

The $\Delta I = 1/2$ Rule in Kaon Decays: A New Look

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Abstract

The $K \rightarrow \pi\pi$ decay amplitudes are studied within the framework of generalized factorization in which the effective Wilson coefficients are gauge-invariant, renormalization-scale and -scheme independent while factorization is applied to the tree-level hadronic matrix elements. Nonfactorized contributions to the hadronic matrix elements of $(V - A)(V - A)$ four-quark operators, which are needed to account for the suppression of the $\Delta I = 3/2$ $K \rightarrow \pi\pi$ amplitude A_2 and the enhancement of the $\Delta I = 1/2$ A_0 amplitude, are phenomenologically extracted from the measured $K^+ \rightarrow \pi^+\pi^0$ decay and found to be large. The A_0/A_2 ratio is predicted to lie in the range 15-17 for $m_s(1 \text{ GeV}) = (127 - 150) \text{ MeV}$. Vertex and penguin-type radiative corrections to the matrix elements of four-quark operators and nonfactorized effects due to soft-gluon exchange account for the bulk of the $\Delta I = 1/2$ rule. Comparison of the present analysis with the chiral-loop approach is given.

I. INTRODUCTION

The effective Hamiltonian approach is the standard starting point for describing the nonleptonic weak decays of hadrons. In this approach, the decay amplitude has the form $A \sim \sum c_i(\mu) \langle O_i(\mu) \rangle$, where the renormalization scale μ separates the short-distance contributions contained in the Wilson coefficient functions $c_i(\mu)$ and the long-distance contributions contained in the hadronic matrix elements of 4-quark operators $\langle O_i(\mu) \rangle$. Of course, the physical amplitude should be independent of the choice of the renormalization scale and scheme. This means that the matrix elements have to be evaluated in the same renormalization scheme as that for Wilson coefficients and renormalized at the same scale μ . In principle, the scale μ can be arbitrary as long as it is large enough to allow for a perturbative calculation of Wilson coefficients. In practice, it is more convenient to choose μ to be the scale of the hadron mass of the decaying particle so that the logarithmic term in the matrix element $\langle O(\mu) \rangle$, which is of order $\ln(M^2/\mu^2)$ with M being the hadron mass, is as small as possible, leaving the large logarithms, which are summed to all orders via the renormalization group technique, to $c(\mu)$.

Since the hadronic matrix elements are very difficult to calculate, it is not surprising that the issue of their μ dependence is generally not addressed in the literature. For meson decays, a popular approach is to evaluate the matrix elements under the factorization hypothesis so that $\langle O(\mu) \rangle$ is factorized into the product of two matrix elements of single currents, governed by decay constants and form factors. However, the information of the scale dependence of $\langle O(\mu) \rangle$ is lost in the factorization approximation because the vector or axial-vector current is partially conserved and hence scale independent. Consequently, the μ dependence of Wilson coefficients does not get compensation from the matrix elements. Although the correct μ dependence of $\langle O(\mu) \rangle$ should be restored by the nonfactorized contributions to hadronic matrix elements, the difficulty is that nonfactorized effects are not amenable owing to their nonperturbative nature. Hence, the question is can we apply factorization to the matrix elements and in the meantime avoid the scale problem with $\langle O(\mu) \rangle$?

Fortunately, the μ dependence of hadronic matrix elements is calculable in perturbation theory. After extracting the μ dependence of $\langle O(\mu) \rangle$ and combining it with the Wilson coefficients $c(\mu)$, we obtain renormalization-scale and -scheme independent effective Wilson coefficients. Then the factorization approximation can be safely applied afterwards to the matrix elements of the operator O_i at the tree level.

For the case of kaon decays such as $K \rightarrow \pi\pi$, it is obvious that μ cannot be chosen to be of order m_K ; instead it has to be at the scale of 1 GeV or larger so that $c(\mu)$ can be reliably computed. Conventionally, the μ dependence of matrix elements involving kaons and pions are calculated by considering chiral loop corrections to $\langle O \rangle$. When chiral loops are regularized using the same dimensional regularization scheme as that for Wilson coefficients, the μ dependence of long-distance contributions will presumably match the scale dependence of Wilson coefficients so that the resulting physical amplitude is μ independent. While the scale dependence of $K \rightarrow \pi\pi$ matrix elements can be furnished by meson loops, it is clear that this approach based on chiral perturbation theory is not applicable to heavy meson decays. Therefore, it is desirable to consider the nonleptonic decays of kaons and heavy mesons within

the same framework of generalized factorization in which the effective Wilson coefficients c_i^{eff} are renormalization-scale and -scheme independent while factorization is applied to the tree-level hadronic matrix elements. The purpose of the present analysis is to see if our understanding of the $\Delta I = 1/2$ rule can be improved in the effective Hamiltonian approach.

The celebrated $\Delta I = 1/2$ rule in kaon decays still remains an enigma after the first observation more than four decades ago. The tantalizing puzzle is the problem of how to enhance the A_0/A_2 ratio of the $\Delta I = 1/2$ to $\Delta I = 3/2$ $K \rightarrow \pi\pi$ amplitudes from the outrageously small value 0.9 [see Eq. (3.32) below] to the observed value 22.2 ± 0.1 (for a review of the $\Delta I = 1/2$ rule, see [1]). In the approach of the effective weak Hamiltonian, the A_0/A_2 ratio is at most of order 7 even after the nonfactorized soft-gluon effects are included [1]. Moreover, the μ dependence of hadronic matrix elements is not addressed in the conventional calculation. In the past ten years or so, most efforts are devoted to computing the matrix elements to $\mathcal{O}(p^4)$ in chiral expansion. This scenario has the advantages that chiral loops provide the necessary scale dependence for hadronic matrix elements and that meson loop contributions to the A_0 amplitude are large enough to accommodate the data. However, it also becomes clear that chiral loops alone cannot explain the A_2 amplitude (see Sec. IV). Consequently, it is necessary to take into account nonfactorized effects on $K^+ \rightarrow \pi^+\pi^0$ in order to have an additional suppression for the $\Delta I = 3/2$ transition.

Contrary to the chiral approach, the difficulty with the μ dependence of the physical $K \rightarrow \pi\pi$ amplitude is circumvented in the present analysis by working in the effective Hamiltonian approach in which the effective Wilson coefficients are gauge-invariant, renormalization-scale and -scheme independent. This approach is not only much simpler than chiral loop calculations but also applicable to heavy meson decays. By extracting nonfactorized effects from $K^+ \rightarrow \pi^+\pi^0$, we shall see that nonperturbative effects due to soft-gluon exchange and perturbative radiative corrections to four-quark operators account for the bulk of the observed $\Delta I = 1/2$ amplitude.

The present paper is organized as follows. In Sec. II we construct scheme and scale independent effective Wilson coefficients relevant to kaon decays. The $K \rightarrow \pi\pi$ matrix elements are evaluated in Sec. III. Comparison of the present analysis of the $\Delta I = 1/2$ rule with the chiral loop approach is made in Sec. IV. Sec. V is for the conclusion.

II. FRAMEWORK

The effective Hamiltonian relevant to $K \rightarrow \pi\pi$ transition is

$$\mathcal{H}_{\text{eff}}(\Delta S = 1) = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(\sum_{i=1}^{10} c_i(\mu) O_i(\mu) \right) + \text{h.c.}, \quad (2.1)$$

where

$$c_i(\mu) = z_i(\mu) + \tau y_i(\mu), \quad (2.2)$$

with $\tau = -V_{td}V_{ts}^*/(V_{ud}V_{us}^*)$, and

$$\begin{aligned}
O_1 &= (\bar{u}d)_{V-A}(\bar{s}u)_{V-A}, & O_2 &= (\bar{u}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \\
O_{3(5)} &= (\bar{s}d)_{V-A} \sum_q (\bar{q}q)_{V-A(V+A)}, & O_{4(6)} &= (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A(V+A)}, \\
O_{7(9)} &= \frac{3}{2}(\bar{s}d)_{V-A} \sum_q e_q (\bar{q}q)_{V+A(V-A)}, & O_{8(10)} &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A(V-A)},
\end{aligned} \tag{2.3}$$

with O_3 – O_6 being the QCD penguin operators, O_7 – O_{10} the electroweak penguin operators and $(\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$. The sum in Eq. (2.3) is over light flavors, $q = u, d, s$. It is obvious that only the Wilson coefficients z_i are relevant to our purposes, as we are only interested in the CP-conserving part of $K \rightarrow \pi\pi$ transitions.

In order to apply the factorization approximation to hadronic matrix elements, we need to compute the μ dependence of matrix elements arising from vertex and penguin-type radiative corrections to four-quark operators and combine it with the Wilson coefficients to form renormalization-scale and -scheme independent effective Wilson coefficient functions (for details, see [2,3]):

$$\begin{aligned}
z_1^{\text{eff}} &= z_1(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{1i} z_i(\mu), \\
z_2^{\text{eff}} &= z_2(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{2i} z_i(\mu), \\
z_3^{\text{eff}} &= z_3(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{3i} z_i(\mu) - \frac{\alpha_s}{24\pi} (C_t + C_p), \\
z_4^{\text{eff}} &= z_4(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{4i} z_i(\mu) + \frac{\alpha_s}{8\pi} (C_t + C_p), \\
z_5^{\text{eff}} &= z_5(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{5i} z_i(\mu) - \frac{\alpha_s}{24\pi} (C_t + C_p), \\
z_6^{\text{eff}} &= z_6(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{6i} z_i(\mu) + \frac{\alpha_s}{8\pi} (C_t + C_p), \\
z_7^{\text{eff}} &= z_7(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{7i} z_i(\mu) + \frac{\alpha}{8\pi} C_e, \\
z_8^{\text{eff}} &= z_8(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{8i} z_i(\mu), \\
z_9^{\text{eff}} &= z_9(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{9i} z_i(\mu) + \frac{\alpha}{8\pi} C_e, \\
z_{10}^{\text{eff}} &= z_{10}(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma_V^{(0)T} \ln \frac{\mu_f}{\mu} + \hat{r}_V^T \right)_{10i} z_i(\mu),
\end{aligned} \tag{2.4}$$

where the superscript T denotes a transpose of the matrix, the anomalous dimension matrix $\gamma_V^{(0)}$ as well as the constant matrix \hat{r}_V arise from the vertex corrections to the operators $O_1 - O_{10}$, C_t , C_p and C_e from the QCD penguin-type diagrams of the operators $O_{1,2}$, the QCD penguin-type diagrams of the operators $O_3 - O_6$, and the electroweak penguin-type diagram of $O_{1,2}$, respectively:

$$\begin{aligned}
C_t &= \tilde{G}(m_u)z_1, \\
C_p &= [\tilde{G}(m_s) + \tilde{G}(m_d)]z_3 + \sum_{i=u,d,s} \tilde{G}(m_i)(z_4 + z_6), \\
C_e &= \frac{8}{9}\tilde{G}(m_u)(z_1 + 3z_2), \\
\tilde{G}(m_q) &= \frac{2}{3}\kappa - G(m_q, k, \mu),
\end{aligned} \tag{2.5}$$

with κ being a parameter characterizing the γ_5 scheme dependence in dimensional regularization, for example,

$$\kappa = \begin{cases} 1 & \text{NDR,} \\ 0 & \text{HV,} \end{cases} \tag{2.6}$$

in the naive dimensional regularization (NDR) and 't Hooft-Veltman (HV) schemes for γ_5 . The function $G(m, k, \mu)$ in Eq. (2.5) is given by

$$G(m, k, \mu) = -4 \int_0^1 dx x(1-x) \ln \left(\frac{m^2 - k^2 x(1-x)}{\mu^2} \right), \tag{2.7}$$

where k^2 is the momentum squared carried by the virtual gluon.

The matrix \hat{r} in (2.4) gives momentum-independent constant terms which depend on the treatment of γ_5 in dimensional regularization. An early evaluation of \hat{r} is performed in the off-shell quark scheme [4]. However, it was pointed out by Buras and Silvestrini [5] that z_i^{eff} thus constructed suffer from gauge and infrared ambiguities since an off-shell external quark momentum, which is usually chosen to regulate the infrared divergence occurred in the radiative corrections to the local 4-quark operators, will introduce a gauge dependence. It was shown recently in [2] that the above-mentioned problems on gauge dependence and infrared singularity connected with the effective Wilson coefficients can be resolved by perturbative QCD (PQCD) factorization theorem. In this formalism, partons, *i.e.*, external quarks, are assumed to be on shell, and both ultraviolet and infrared divergences in radiative corrections are isolated using the dimensional regularization. Because external quarks are on shell, gauge invariance of the decay amplitude is maintained under radiative corrections to all orders. This statement is confirmed by an explicit one-loop calculation in [2]. The obtained ultraviolet poles are subtracted in a renormalization scheme, while the infrared poles are absorbed into universal nonperturbative bound-state wave functions. Explicitly, the effective Wilson coefficient has the generic expression

$$c^{\text{eff}} = c(\mu)g_1(\mu)g_2(\mu_f), \tag{2.8}$$

where $g_1(\mu)$ is an evolution factor from the scale μ to m_Q , whose anomalous dimension is the same as that of $c(\mu)$, and $g_2(\mu_f)$ describes the evolution from m_Q to μ_f (μ_f being a factorization scale arising from the dimensional regularization of infrared divergences), whose anomalous dimension differs from that of $c(\mu)$ because of the inclusion of the dynamics associated with spectator quarks. For kaon decays under consideration, there is no any heavy quark mass scale between m_c and m_K . Hence, the logarithmic term emerged in the vertex

corrections to 4-quark operators is of the form $\ln \mu_f/\mu$ as shown in Eq. (2.4). We will set $\mu_f = 1$ GeV in order to have a reliable estimate of perturbative effects on effective Wilson coefficients.

The scale dependence of vertex and penguin-type corrections shown in Eq. (2.4) is governed by the terms $\gamma_V^{(0)} \ln \mu$ and $\tilde{G}(\mu)$, while the γ_5 -scheme dependence is determined by the matrix \hat{r}_V as well as $\tilde{G}(\kappa)$. Formally, one can show that the μ and γ_5 -scheme dependence of the next-to-leading order (NLO) Wilson coefficient, say $z_1(\mu)$, is compensated by $\gamma_V^{(0)} \ln \mu$ and \hat{r}_V , respectively, to the order of α_s [2,3]. This means that the NLO Wilson coefficients $z_i(\mu)$ appearing in Eq. (2.4) together with $\alpha_s/4\pi$ or $\alpha/4\pi$ should be replaced by the lowest-order values $z_i^{\text{LO}}(\mu)$. The numerical values of z_i^{eff} are displayed in Table I. We see that except for $z_{6,7}^{\text{eff}}$, effective Wilson coefficients shown in the last two columns of Table I are indeed renormalization scheme independent, as it should be.

TABLE I. $\Delta S = 1$ Wilson coefficients at $\mu = 1$ GeV for $m_t = 170$ GeV and $\Lambda_{\overline{\text{MS}}}^{(4)} = 325$ MeV, taken from Table XVIII of [6]. Also shown are the effective Wilson coefficients obtained from $z_i^{\text{NDR}}(\mu)$ and $z_i^{\text{HV}}(\mu)$ via Eq. (2.4) with $\mu = 1$ GeV, $\mu_f = 1$ GeV and $k^2 = m_K^2/2$.

	LO	NDR	HV	$z_i^{\text{eff}}(\text{NDR})$	$z_i^{\text{eff}}(\text{HV})$
z_1	1.433	1.278	1.371	1.718	1.713
z_2	-0.748	-0.509	-0.640	-1.113	-1.110
z_3	0.004	0.013	0.007	0.034	0.033
z_4	-0.012	-0.035	-0.017	-0.088	-0.087
z_5	0.004	0.008	0.004	0.026	0.026
z_6	-0.013	-0.035	-0.014	-0.093	-0.089
z_7/α	0.008	0.011	-0.002	0.063	0.069
z_8/α	0.001	0.014	0.010	0.016	0.013
z_9/α	0.008	0.018	0.005	0.072	0.078
z_{10}/α	-0.001	-0.008	-0.010	-0.011	-0.012

In the late 70's and early 80's, it had been suggested that penguin operators may account for the $\Delta I = 1/2$ rule observed in kaon decays. With the advent of the effective Hamiltonian approach, it is realized that the $\Delta I = 1/2$ selection rule cannot be dominated by the penguin mechanism. One popular argument is that at the scale, say $\mu \gtrsim m_c$, the penguin Wilson coefficients are negligible due to the (incomplete) GIM mechanism. Since the penguin Wilson coefficients become important at $\mu = 1$ GeV, for instance, one may wonder if the physical penguin contributions to $K \rightarrow \pi\pi$ is independent of the choice of μ . The point is that although z_3, \dots, z_{10} vanish at, say $\mu = 2$ GeV, the effect of the penguin diagrams with the internal u quark induced by the current-current operator O_1 has to be taken into account when evaluating matrix elements. Consequently, the total penguin contribution is scale independent, and this is the merit of effective Wilson coefficients in which the perturbative effect of the penguin diagram with the internal u quark is already included.

We add a remark before ending this section. A dynamical phase can arise from the time-like penguin diagram involving internal u loop quarks. However, since this penguin-induced phase is incorporated into the isospin zero final-state interaction phase shift δ_0 to be introduced below in Eq. (3.11), its contribution should not be double-counted in the effective Wilson coefficients.

III. CALCULATIONS

In this section we first study the $K \rightarrow \pi\pi$ matrix elements based on the vacuum insertion approximation, and then turn to the $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes.

A. Matrix Elements

It is convenient to make isospin decomposition of the matrix elements $\langle O_i \rangle_{0,2} \equiv \langle \pi\pi, I = 0, 2 | O_i | K^0 \rangle$ which are related to $K - \pi\pi$ transitions via

$$\begin{aligned}\langle O_i \rangle_0 &= \frac{1}{\sqrt{6}} \left(2\langle \pi^+ \pi^- | O_i | K^0 \rangle + \langle \pi^0 \pi^0 | O_i | K^0 \rangle \right), \\ \langle O_i \rangle_2 &= \frac{1}{\sqrt{3}} \left(\langle \pi^+ \pi^- | O_i | K^0 \rangle - \langle \pi^0 \pi^0 | O_i | K^0 \rangle \right) = \sqrt{\frac{2}{3}} \langle \pi^+ \pi^0 | O_i | K^+ \rangle.\end{aligned}\quad (3.1)$$

Conventionally, the matrix elements $\langle O_i \rangle_{0,2}$ are evaluated using the vacuum insertion approximation (i.e. factorization hypothesis). Under this assumption, we have, for example,

$$\begin{aligned}\langle \pi^+ \pi^- | O_1 | K^0 \rangle &= \langle \pi^+ | (\bar{u}d) | 0 \rangle \langle \pi^- | (\bar{s}u) | K^0 \rangle + \frac{1}{N_c} \langle \pi^+ \pi^- | (\bar{u}u) | 0 \rangle \langle 0 | (\bar{s}d) | K^0 \rangle \\ &= f_\pi (m_K^2 - m_\pi^2) F_0^{K\pi}(m_\pi^2),\end{aligned}\quad (3.2)$$

where the form factor F_0 is defined in [7] and the W -exchange contribution vanishes due to vector current conservation. The q^2 dependence of the form factor F_0 is usually assumed to be dominated by near poles in a monopole manner:

$$F_0^{K\pi}(q^2) = \frac{F_0^{K\pi}(0)}{1 - \frac{q^2}{m_*^2}} \approx F_0^{K\pi}(0) \left(1 + \frac{q^2}{m_*^2} \right), \quad (3.3)$$

where m_* is the pole mass of the 0^+ scalar meson with the quantum number of $s\bar{q}$ ($q = u, d$). In chiral perturbation theory (ChPT), we have $F_0^{K\pi}(0) = 1$ due to vector current conservation and (see [1] for details)

$$\frac{1}{m_*^2} = \frac{8L_5}{f_\pi^2} = \left(\frac{f_K}{f_\pi} - 1 \right) \frac{1}{(m_K^2 - m_\pi^2)} \approx \frac{1}{\Lambda_\chi^2}, \quad (3.4)$$

where L_5 is one of the coupling constants in the $\mathcal{O}(p^4)$ chiral Lagrangian for strong interactions, and $\Lambda_\chi \approx 2\pi f_\pi$ (our $f_\pi = 132$ MeV) is the chiral-symmetry breaking scale [8]. Therefore,

$$\langle \pi^+ \pi^- | O_1 | K^0 \rangle = f_\pi (m_K^2 - m_\pi^2) \left(1 + \frac{m_\pi^2}{\Lambda_\chi^2} \right). \quad (3.5)$$

In ChPT, the term proportional to m_π^2/Λ_χ^2 is counted as a contribution of $\mathcal{O}(p^4)$.

Contrary to charmless B decays, the penguin operators $O_{5,6}$, do not contribute directly to $K^0 \rightarrow \pi\pi$ because of the wave function $\pi^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$ and the SU(3)-singlet nature of the $V + A$ current. Nevertheless, the Fierz transformation of $O_{5,6}$ via $(V - A)(V + A) \rightarrow -2(S + P)(S - P)$ does make contributions. For example,

$$\begin{aligned} \langle \pi^+ \pi^- | O_6 | K^0 \rangle &= \frac{2}{3} \langle \pi^+ | \bar{u} \gamma_5 d | 0 \rangle \langle \pi^- | \bar{s} u | K^0 \rangle - \frac{2}{3} \langle \pi^+ \pi^- | \bar{d} d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K^0 \rangle \\ &\quad + \frac{2}{3} (\langle \bar{d} d \rangle + \langle \bar{s} s \rangle) \langle \pi^+ \pi^- | \bar{s} \gamma_5 d | K^0 \rangle. \end{aligned} \quad (3.6)$$

The matrix elements of scalar and pseudoscalar densities can be evaluated using equations of motion or the chiral representation of quark densities. The former method gives

$$\begin{aligned} \langle \pi^-(q) | \bar{s} u | K^0(k) \rangle &= v \left[1 + \frac{(k - q)^2}{\Lambda_\chi^2} \right], & \langle \pi^+(q) | \bar{u} \gamma_5 d | 0 \rangle &= i f_\pi v, \\ \langle \pi^+(q_+) \pi^-(q_-) | \bar{d} d | 0 \rangle &= v \left[1 + \frac{(q_+ + q_-)^2}{\Lambda_\chi^2} \right], & \langle 0 | \bar{s} \gamma_5 d | K^0(k) \rangle &= i f_K v, \end{aligned} \quad (3.7)$$

where uses of Eqs. (3.3) and (3.4) have been made, and

$$v = \frac{m_{\pi^\pm}^2}{m_u + m_d} = \frac{m_{K^0}^2}{m_d + m_s} = \frac{m_K^2 - m_\pi^2}{m_s - m_u} \quad (3.8)$$

characterizes the quark-order parameter $\langle \bar{q} q \rangle$ which breaks chiral symmetry spontaneously. The second term on the r.h.s. of Eq. (3.6) is the so-called spacelike penguin contribution. Unlike the case of hadronic charmless B decays, the spacelike penguin diagram in $K \rightarrow \pi\pi$ is calculable. The last term in Eq. (3.6), which is a tadpole contribution arising from the vacuum expectational values of quark bilinears, does not contribute to the physical $K \rightarrow \pi\pi$ amplitude [1]. Hence, we obtain

$$\langle \pi^+(q_+) \pi^-(q_-) | O_6 | K^0(k) \rangle = -i \frac{4}{3} f_\pi v^2 \frac{k^2 - q_+^2}{\Lambda_\chi^2} + \mathcal{O} \left(\frac{1}{\Lambda_\chi^4} \right), \quad (3.9)$$

where we have applied Eq. (3.4).

The matrix elements obtained under the vacuum insertion approximation are summarized below:

$$\begin{aligned} \langle O_1 \rangle_0 &= \frac{1}{3} X \left(2 - \frac{1}{N_c} \right), & \langle O_1 \rangle_2 &= \frac{\sqrt{2}}{3} X \left(1 + \frac{1}{N_c} \right), \\ \langle O_2 \rangle_0 &= \frac{1}{3} X \left(-1 + \frac{2}{N_c} \right), & \langle O_2 \rangle_2 &= \frac{\sqrt{2}}{3} X \left(1 + \frac{1}{N_c} \right), \\ \langle O_3 \rangle_0 &= \frac{1}{N_c} X, & \langle O_4 \rangle_0 &= X, \end{aligned}$$

$$\begin{aligned}
\langle O_5 \rangle_0 &= -\frac{4}{N_c} \sqrt{\frac{3}{2}} v^2 (f_K - f_\pi), & \langle O_6 \rangle_0 &= -4 \sqrt{\frac{3}{2}} v^2 (f_K - f_\pi), \\
\langle O_7 \rangle_0 &= \frac{\sqrt{6}}{N_c} f_K v^2 + \frac{1}{2} X, & \langle O_7 \rangle_2 &= \frac{\sqrt{3}}{N_c} f_\pi v^2 - \frac{1}{\sqrt{2}} X, \\
\langle O_8 \rangle_0 &= \sqrt{6} f_K v^2 + \frac{1}{2 N_c} X, & \langle O_8 \rangle_2 &= \sqrt{3} f_\pi v^2 - \frac{1}{N_c \sqrt{2}} X, \\
\langle O_9 \rangle_0 &= -\frac{1}{2} X \left(1 - \frac{1}{N_c} \right), & \langle O_9 \rangle_2 &= -\frac{1}{\sqrt{2}} X \left(1 + \frac{1}{N_c} \right), \\
\langle O_{10} \rangle_0 &= \frac{1}{2} X \left(1 - \frac{1}{N_c} \right), & \langle O_{10} \rangle_2 &= \frac{1}{\sqrt{2}} X \left(1 + \frac{1}{N_c} \right),
\end{aligned} \tag{3.10}$$

where $X = \sqrt{3/2} f_\pi (m_K^2 - m_\pi^2)$ and $1/\Lambda_\chi^2$ corrections to $(V - A)(V \pm A)$ matrix elements as well as $1/\Lambda_\chi^4$ corrections to $(S + P)(S - P)$ matrix elements have been neglected.

In terms of the isospin matrix elements, the corresponding isospin decay amplitudes are given by

$$\begin{aligned}
A_I e^{i\delta_I} &= \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i c_i(\mu) \langle O_i(\mu) \rangle_I, \\
&= \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i c_i^{\text{eff}} \langle O_i \rangle_I,
\end{aligned} \tag{3.11}$$

and hence

$$\text{Re } A_{0,2} = \frac{G_F}{\sqrt{2}} \frac{V_{ud} V_{us}^*}{\cos \delta_{0,2}} \sum_i z_i^{\text{eff}} \langle O_i \rangle_{0,2}, \tag{3.12}$$

where δ_0 and δ_2 are S-wave $\pi\pi$ scattering isospin phase shifts. In the present paper, we will use the analysis of [9] for phase shifts:

$$\delta_0 = (34.2 \pm 2.2)^\circ, \quad \delta_2 = -(6.9 \pm 0.2)^\circ. \tag{3.13}$$

Experimentally, the isospin $K \rightarrow \pi\pi$ amplitudes are given by [1]

$$\text{Re } A_0 = 3.323 \times 10^{-7} \text{ GeV}, \quad \text{Re } A_2 = 1.497 \times 10^{-8} \text{ GeV}. \tag{3.14}$$

B. The $\Delta I = 3/2$ amplitude

It is straightforward to show from Eqs. (3.10) and (3.12) that the isospin 2 amplitude of $K \rightarrow \pi\pi$ has the form

$$A_2^{(0)} = \frac{G_F}{\sqrt{2}} \frac{V_{ud} V_{us}^*}{\cos \delta_2} \left\{ \left[a_1 + a_2 + \frac{3}{2} (-a_7 + a_9 + a_{10}) \right] \frac{\sqrt{2}}{3} X + \sqrt{3} f_\pi v^2 a_8 \right\}, \tag{3.15}$$

where

$$a_{2i} = z_{2i}^{\text{eff}} + \frac{1}{N_c} z_{2i-1}^{\text{eff}}, \quad a_{2i-1} = z_{2i-1}^{\text{eff}} + \frac{1}{N_c} z_{2i}^{\text{eff}} \tag{3.16}$$

for $i = 1, \dots, 5$, and the superscript (0) indicates that this amplitude is induced by pure $\Delta I = 3/2$ weak interactions. Since the electroweak penguin coefficients are very small compared to z_1^{eff} and z_2^{eff} , it is clear that the $\Delta I = 3/2$ decay amplitude is entirely governed by current-current 4-quark operators. It is also known that $K^+ \rightarrow \pi^+\pi^0$ (or $\Delta I = 3/2$ $K^0 \rightarrow \pi\pi$ decays) can be generated from the $\Delta I = 1/2$ decays $K^+ \rightarrow \pi^+\eta(\eta')$ followed by the isospin breaking mixing $\pi^0 - \eta - \eta'$ [10,11]. As a result, the total $\Delta I = 3/2$ amplitude reads

$$A_2 = \frac{A_2^{(0)}}{1 - \Omega_{\text{IB}}}, \quad (3.17)$$

where the expression of $\Omega_{\text{IB}} \equiv A_2^{\text{IB}}/A_2$ can be found in Appendix A. Employing the quark mass ratios $m_d/m_u = 0.553 \pm 0.043$ and $m_s/m_d = 18.9 \pm 0.8$ obtained in a recent detailed analysis based on ChPT [12], we find from Eq. (A7) that

$$\Omega_{\text{IB}} = 0.25 \pm 0.02. \quad (3.18)$$

Eqs. (3.15)-(3.18) lead to

$$A_2 = 4.133 (z_1^{\text{eff}} + z_2^{\text{eff}}) \left(1 + \frac{1}{N_c}\right) \times 10^{-8} \text{GeV}. \quad (3.19)$$

Using the effective Wilson coefficients z_i^{eff} given in Table I, it is easily seen that the predicted A_2 is too large by a factor of 2.2 compared to experiment (3.14). This means that nonfactorized contributions that have been neglected thus far should be taken into account. For $K \rightarrow \pi\pi$ decays, nonfactorizable effects in hadronic matrix elements can be absorbed into the parameters a_i^{eff} [13–15]:

$$a_{2i}^{\text{eff}} = z_{2i}^{\text{eff}} + \left(\frac{1}{N_c} + \chi_{2i}\right) z_{2i-1}^{\text{eff}}, \quad a_{2i-1}^{\text{eff}} = z_{2i-1}^{\text{eff}} + \left(\frac{1}{N_c} + \chi_{2i-1}\right) z_{2i}^{\text{eff}}, \quad (3.20)$$

where the nonfactorized terms $\chi_{1,2}$ relevant to $K^+ \rightarrow \pi^+\pi^0$ decay are given by [15–17]

$$\chi_1 = \varepsilon_8^{(K^+\pi^0, \pi^+)} + \frac{a_1}{z_2^{\text{eff}}} \varepsilon_1^{(K^+\pi^0, \pi^+)}, \quad \chi_2 = \varepsilon_8^{(K^+\pi^+, \pi^0)} + \frac{a_2}{z_1^{\text{eff}}} \varepsilon_1^{(K^+\pi^+, \pi^0)}, \quad (3.21)$$

with $a_{1,2} = z_{1,2}^{\text{eff}} + z_{2,1}^{\text{eff}}/N_c$, and

$$\begin{aligned} \varepsilon_1^{(K^+\pi^0, \pi^+)} &= \frac{\langle \pi^+\pi^0 | (\bar{u}d)_{V-A} (\bar{s}u)_{V-A} | K^+ \rangle_{\text{nf}}}{\langle \pi^+\pi^0 | (\bar{u}d)_{V-A} (\bar{s}u)_{V-A} | K^+ \rangle_{\text{f}}} = \frac{\langle \pi^+\pi^0 | (\bar{u}d)_{V-A} (\bar{s}u)_{V-A} | K^+ \rangle}{\langle \pi^+ | (\bar{u}d)_{V-A} | 0 \rangle \langle \pi^+ | (\bar{s}u)_{V-A} | K^+ \rangle} - 1, \\ \varepsilon_8^{(K^+\pi^0, \pi^+)} &= \frac{1}{2} \frac{\langle \pi^+\pi^0 | (\bar{u}\lambda^a d)_{V-A} (\bar{s}\lambda^a u)_{V-A} | K^+ \rangle}{\langle \pi^+ | (\bar{u}d)_{V-A} | 0 \rangle \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle}, \end{aligned} \quad (3.22)$$

being nonfactorizable terms originated from color-singlet and color-octet currents, respectively, and $(\bar{q}_1 \lambda^a q_2)_{V-A} \equiv \bar{q}_1 \lambda^a \gamma_\mu (1 - \gamma_5) q_2$. The subscripts ‘f’ and ‘nf’ in Eq. (3.22) stand for factorizable and nonfactorizable contributions, respectively, and the superscript $(K^+\pi^0, \pi^+)$ in Eq. (3.21) means that the π^+ is factored out in the factorizable amplitude of $K^+ \rightarrow \pi^+\pi^0$ and likewise for the superscript $(K^+\pi^+, \pi^0)$. In the large- N_c limit, $\varepsilon_1 = \mathcal{O}(1/N_c^2)$ and

$\varepsilon_8 = \mathcal{O}(1/N_c)$ [17]. Therefore, the nonfactorizable term χ in the $N_c \rightarrow \infty$ limit is dominated by color octet-octet operators.

Assuming $\chi_1 = \chi_2$ in Eq. (3.20) for a_1^{eff} and a_2^{eff} and fitting Eq. (3.19), in which $1/N_c$ is replaced by $1/N_c + \chi$, to the experimental value (3.14), we obtain*

$$\chi(K \rightarrow \pi\pi) = -0.73, \quad (3.23)$$

and hence

$$a_1^{\text{eff}} = 2.16, \quad a_2^{\text{eff}} = -1.80. \quad (3.24)$$

For comparison, the nonfactorized effects in hadronic two-body decays of charmed and bottom mesons are given by [18]

$$\chi_2(D \rightarrow \bar{K}\pi) \sim -0.33, \quad \chi_2(B \rightarrow D\pi) \sim (0.12 - 0.21). \quad (3.25)$$

The fact that

$$|\chi(K \rightarrow \pi\pi)| \gg |\chi_2(D \rightarrow \bar{K}\pi)| \gg |\chi_2(B \rightarrow D\pi)| \quad (3.26)$$

is consistent with the intuitive picture that soft gluon effects become stronger when final-state particles move slower, allowing more time for significant final-state interactions after hadronization [13].

Note that in B or D decays, the parameters $a_{1,2}^{\text{eff}}$ and hence $\chi_{1,2}$ in principle can be determined separately from experiments under some plausible assumptions. For example, $\chi_1(D \rightarrow \bar{K}\pi)$ and $\chi_2(D \rightarrow \bar{K}\pi)$ can be extracted from the isospin analysis of $D^0 \rightarrow K^-\pi^+$, $\bar{K}^0\pi^0$ and $D^+ \rightarrow \bar{K}^0\pi^+$ data provided that the W -exchange is negligible.[†] By contrast, $\chi_1(K \rightarrow \pi\pi)$ and $\chi_2(K \rightarrow \pi\pi)$ cannot be determined from the data without invoking a further assumption because neutral $K^0 \rightarrow \pi\pi$ decays receive additional penguin contributions. That is why we make the universality assumption $\chi_1 = \chi_2$ to extract $a_{1,2}(K \rightarrow \pi\pi)$ from the measurement of $K^+ \rightarrow \pi^+\pi^0$.

In the literature, the effective parameters a_i^{eff} are sometimes expressed in terms of the the scheme- and scale-dependent Wilson coefficients $z_i(\mu)$, for example,

$$a_1^{\text{eff}} = z_1(\mu) + \left(\frac{1}{N_c} + \tilde{\chi}_1(\mu)\right) z_2(\mu), \quad a_2^{\text{eff}} = z_2(\mu) + \left(\frac{1}{N_c} + \tilde{\chi}_2(\mu)\right) z_1(\mu), \quad (3.27)$$

where we have put a tilde on $\chi_{1,2}$ to distinguish them from $\chi_{1,2}$ defined in Eq. (3.20). Then it is clear that $\tilde{\chi}_{1,2}$ must be γ_5 -scheme and scale dependent in order to ensure the scheme and scale independence of $a_{1,2}^{\text{eff}}$. Comparing Eqs. (3.27) and (3.20), we see that $\tilde{\chi}_{1,2}$ receive contributions from vertex radiative corrections. It should be stressed that the assumption

*In the so-called large- N_c approach, one has $\chi = -1/3$ to the leading $1/N_c$ expansion.

[†]Since in general $|z_1/z_2| \gg 1$, the determination of χ_2 is easier and more reliable than χ_1 .

$\tilde{\chi}_1 = \tilde{\chi}_2$ cannot lead to γ_5 -scheme independent $a_{1,2}^{\text{eff}}$. To see this, we assume (3.24) to be the true values for $a_{1,2}^{\text{eff}}$ and apply the Wilson coefficients evaluated at $\mu = 1$ GeV in NDR and HV schemes given in Table I. We find

$$\begin{aligned}\tilde{\chi}_1^{\text{NDR}}(\mu) &= -2.07, & \tilde{\chi}_2^{\text{NDR}}(\mu) &= -1.34, \\ \tilde{\chi}_1^{\text{HV}}(\mu) &= -1.57, & \tilde{\chi}_2^{\text{HV}}(\mu) &= -1.18,\end{aligned}\tag{3.28}$$

at $\mu = 1$ GeV by fitting (3.27) to (3.24). This implies that phenomenologically it is not possible to determine $a_{1,2}^{\text{eff}}$ from the data of $K \rightarrow \pi\pi$ if we start with the scheme- and scale-dependent Wilson coefficients $z_i(\mu)$ without taking into account vertex corrections to $\tilde{\chi}_{1,2}$.

C. The $\Delta I = 1/2$ amplitude

From Eqs. (3.10) and (3.12) we obtain the $\Delta I = 1/2$ amplitude:

$$\begin{aligned}A_0 &= \frac{G_F}{\sqrt{2}} \frac{V_{ud}V_{us}^*}{\cos \delta_0} \left\{ \left[\frac{2}{3}a_1 - \frac{1}{3}a_2 + a_4 + \frac{1}{2}(a_7 - a_9 + a_{10}) \right] X \right. \\ &\quad \left. - 2\sqrt{6}v^2(f_K - f_\pi)a_6 + \sqrt{6}v^2f_Ka_8 \right\},\end{aligned}\tag{3.29}$$

where we have neglected the contribution arising from $\pi^0 - \eta - \eta'$ mixing. For simplicity, we have also dropped the superscript ‘eff’ of the parameters a_i . To incorporate nonfactorized effects, we shall make the universality assumption:

$$\begin{aligned}\chi_{LL} &\equiv \chi_1 = \chi_2 = \chi_3 = \chi_4 = \chi_9 = \chi_{10}, \\ \chi_{LR} &\equiv \chi_5 = \chi_6 = \chi_7 = \chi_8.\end{aligned}\tag{3.30}$$

The nonfactorized effects in the matrix elements of $(V - A)(V + A)$ operators are *a priori* different from that of $(V - A)(V - A)$ operators. Indeed, we have learned from hadronic charmless B decays that $\chi_{LR} \neq \chi_{LL}$ [3]. However, in the absence of information for the nonfactorized contributions to $K \rightarrow \pi\pi$ penguin operators, we shall assume $\chi_{LR} \approx \chi_{LL} = -0.73$ for simplicity. Moreover, we found in actual calculations, A_0 is insensitive to the value of χ_{LR} . From Eq. (3.29) and Table I, it is easily seen that the nonfactorized term $\chi_{LL} = -0.73$, which is needed to suppress A_2 to the observed value, will enhance the tree contribution to A_0 by a factor of 1.9; that is, the tree contribution to A_0/A_2 ratio is increased by a factor of 3 !

Treating the strange quark mass m_s and hence the parameter v as a free parameter, we plot in Fig. 1 the ratio A_0/A_2 as a function of m_s at the renormalization scale $\mu = 1$ GeV. Specifically, we obtain

$$\frac{A_0}{A_2} = \begin{cases} 17.1 & \text{at } m_s(1 \text{ GeV}) = 127 \text{ MeV}, \\ 15.3 & \text{at } m_s(1 \text{ GeV}) = 150 \text{ MeV}. \end{cases}\tag{3.31}$$

It is clear that m_s is favored to be smaller. Presently there is no consensus regarding the values of light quark masses. It is interesting to note that several recent lattice calculations

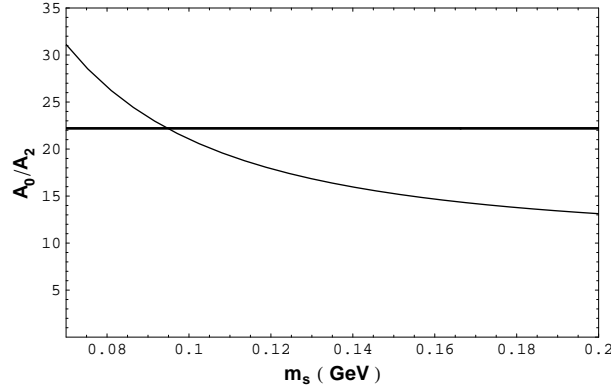


FIG. 1. The ratio of A_0/A_2 versus m_s (in units of GeV) at the renormalization scale $\mu = 1$ GeV. The solid thick line is the experimental value for A_0/A_2 .

give a lighter strange quark mass: Results using the Sheikholeslami-Wohlert fermion yield $m_s = (95 \pm 16)$ MeV [19], a computation based on domain wall fermions obtains $m_s = (95 \pm 26)$ MeV [20], a quenched QCD calculation together with the quark mass ratios from ChPT gives $m_s = (97 \pm 7)$ MeV [21], and a new unquenched lattice result indicates a still lower number $m_s = (84 \pm 7)$ MeV [22], all in the $\overline{\text{MS}}$ scheme at a scale of 2 GeV. The strange quark mass 95 MeV at $\mu = 1$ GeV corresponds to $m_s = 127$ MeV at $\mu = 1$ GeV.

It is instructive to see how the prediction of the $\Delta I = 1/2$ rule progresses at various steps. In the absence of QCD corrections, we have $a_2 = \frac{1}{3}a_1$ and $a_3 = a_4 \cdots = a_{10} = 0$ under the vacuum insertion approximation. It follows from Eqs. (3.15) and (3.29) that [1]

$$\frac{A_0}{A_2} = \frac{5}{4\sqrt{2}} = 0.9 \quad (\text{in absence of QCD corrections}). \quad (3.32)$$

With the inclusion of lowest-order short-distance QCD corrections to the Wilson coefficients z_1 and z_2 evaluated at $\mu = 1$ GeV, A_0/A_2 is enhanced from the value of 0.9 to 2.0, and it becomes 2.4 for $m_s(1 \text{ GeV}) = 127$ MeV when QCD and electroweak penguin effects are included. This ratio is suppressed to 1.8 with the inclusion of the isospin-breaking effect, but it is increased again to the value of 2.1 in the presence of final-state interactions with $\delta_0 = 34.2^\circ$ and $\delta_2 = -6.9^\circ$. Replacing $c_i^{\text{LO}}(\mu)$ by the effective Wilson coefficients c_i^{eff} , or equivalently replacing the LO Wilson coefficients by the NLO ones and including vertex and penguin-type corrections to four-quark operators, we find $A_0/A_2 = 4.8$. Finally, the inclusion of nonfactorized effects on hadronic matrix elements will enhance A_0/A_2 to the value of 17.1. In short, the enhancement of the ratio A_0/A_2 is due to the cumulative effects of the short-distance Wilson coefficients, penguin operators, final-state interactions, nonfactorized effects due to soft-gluon exchange, and radiative corrections to the matrix elements of four-quark operators. Among them, the last two effects, which are usually not addressed in previous studies (in particular, the last one), play an essential role for explaining the bulk of the $\Delta I = 1/2$ rule. In present calculations, penguin operators account for 35% of the $\Delta I = 1/2$ rule for $m_s(1 \text{ GeV}) = 127$ MeV.

Note that thus far we have neglected the W -exchange effect, vanishing in the vacuum insertion approximation. Since the W -exchange amplitude in charmed meson decay is comparable

to the internal W -emission one [23], it is conceivable that in kaon physics the long-distance contribution to W -exchange is as important as the external W -emission amplitude. Therefore, the W -exchange mechanism could provide an additional important enhancement of the A_0/A_2 ratio.

IV. COMPARISON WITH CHIRAL APPROACH

Since the scale dependence of hadronic matrix elements is lost in the factorization approach, it has been advocated that a physical cutoff Λ_c , which is introduced to regularize the quadratic (and logarithmic) divergence of the long-distance chiral loop corrections to $K \rightarrow \pi\pi$ amplitudes, can be identified with the renormalization scale μ of the Wilson coefficients [24]. A most recent calculation along this line which includes $\mathcal{O}(p^4)$ tree contributions [25] indicates that while the isospin amplitude A_0 is largely enhanced, the amplitude A_2 is highly unstable relative to the cutoff scale Λ_c and it even changes sign at $\Lambda_c \gtrsim 650$ MeV [25,26]. The large uncertainty for A_2 arises from the fact that the two numerically leading terms, the tree level and the one-loop quadratically divergent term, have approximately the same size but opposite sign.

Since the scale dependence of Wilson coefficients is of the logarithmic type, it seems quite unnatural to match the quadratic cutoff with the μ dependence of $c(\mu)$. Therefore, it is necessary to use the dimensional regularization to regularize the chiral loop divergences and apply the same renormalization scheme, say the $\overline{\text{MS}}$ scheme, in order to consistently match the scale dependence of Wilson coefficients evaluated using the same regularization scheme. In this case, the inclusion of chiral loops will make a large enhancement for A_0 and a small enhancement for A_2 [27]. However, as stressed in passing, the naive prediction of A_2 in the absence of nonfactorizable effects is too large (by a factor 1.6 in our case) compared to experiment. Therefore, chiral-loop corrections to A_2 will make the discrepancy between theory and experiment even worse. Evidently, this indicates that *not all the long-distance nonfactorized contributions to hadronic matrix elements are fully accounted for by chiral loops*. Several authors [28,26] have considered different models, for instance the chiral quark model or the Nambu-Jona-Lasinio model, to incorporate nonfactorized contributions arising from soft gluonic corrections. For example, the nonfactorized gluonic corrections computed in the chiral quark model amount to replacing $1/N_c$ in the matrix elements $\langle O_{1-3} \rangle_{0,2}$ [see Eq. (3.10)] by [28]

$$\frac{1}{N_c} \rightarrow \frac{1}{N_c}(1 - \delta_{\langle GG \rangle}) \equiv \frac{1}{N_c} \left(1 - \frac{N_c}{2} \frac{4\pi^2 \langle \alpha_s GG / \pi \rangle}{\Lambda_\chi^4} \right), \quad (4.1)$$

parametrized in terms of the gluon condensate $\langle \alpha_s GG / \pi \rangle$. It is clear that the correction $-\delta_{\langle GG \rangle}/3$ plays the same role as the nonfactorized terms χ_i defined in Eq. (3.20). This nonfactorized effect is important since it can suppress the A_2 amplitude. Using $\langle \alpha_s GG / \pi \rangle = (334 \pm 4 \text{ MeV})^4$ [29], one obtains $\delta_{\langle GG \rangle} = 1.51$. It is clear that the soft gluon correction, corresponding to $\chi_1 = \chi_2 = -0.50$, is large enough to revert the sign of the $1/N_c$ term and thus suppress the A_2 amplitude.

In the present analysis, the nonfactorized contribution to the matrix elements of $(V - A)(V - A)$ operators characterized by the nonfactorized terms $\chi_{1,2} = -0.73$ is comparable to that obtained in the chiral quark model. Therefore, in a rough sense, vertex and penguin-type radiative corrections to $K \rightarrow \pi\pi$ matrix elements in the effective Hamiltonian approach corresponds to chiral-loop contributions in the aforementioned chiral approach. However, the penguin contribution to the A_0 amplitude in the latter approach is usually smaller than that in the former. For example, the O_6 operator contribution to A_0 is about 20% in [28], and it is even smaller in other chiral-loop calculations.

V. CONCLUSION

We have studied $K \rightarrow \pi\pi$ decays within the framework of generalized factorization in which the effective Wilson coefficients are renormalization-scale and -scheme independent while factorization is applied to the tree-level hadronic matrix elements. Nonfactorizable contributions to the hadronic matrix elements of $(V - A)(V - A)$ four-quark operators are extracted from the measured $K^+ \rightarrow \pi^+\pi^0$ decay to be $\chi_{1,2} = -0.73$ which explains the suppression of the $\Delta I = 3/2$ $K \rightarrow \pi\pi$ amplitude A_2 and the enhancement of the $\Delta I = 1/2$ A_0 amplitude. The $\Delta I = 1/2$ rule arises from the cumulative effects of the short-distance Wilson coefficients, penguin operators, final-state interactions, nonfactorized effects due to soft-gluon exchange, and radiative corrections to the matrix elements of four-quark operators. In particular, the last two effects are the main ingredients for the large enhancement of A_0 with respect to A_2 . The A_0/A_2 ratio is predicted to lie in the range 15-17 for $m_s(1\text{ GeV}) = (127 - 150)\text{ MeV}$. Comparison of the present analysis of the $\Delta I = 1/2$ rule with the chiral-loop approach is given.

ACKNOWLEDGMENTS

I wish to thank S. Bertolini for useful comments. This work is supported in part by the National Science Council of the Republic of China under Grants No. NSC88-2112-M006-013.

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APPENDIX

A. ISOSPIN BREAKING EFFECTS ON $\Delta I = 3/2$ $K \rightarrow \pi\pi$ AMPLITUDE

In this Appendix we give an updated estimate of isospin breaking contribution to $K^+ \rightarrow \pi^+\pi^0$ due to the $\pi - \eta - \eta'$ mixing. Writing

$$A_2 = A_2^{(0)} + A_2^{\text{IB}}, \quad (\text{A1})$$

we have [11]

$$\begin{aligned} \Omega_{\text{IB}} \equiv \frac{A_2^{\text{IB}}}{A_2} = \frac{1}{3\sqrt{2}} \frac{A_0}{A_2} \frac{m_d - m_u}{m_s} & \left[(\cos\theta - \sqrt{2}\sin\theta)(\cos\theta - \sqrt{2}\frac{\rho}{1+\delta}\sin\theta) \right. \\ & \left. + (\sin\theta + \sqrt{2}\cos\theta)(\sin\theta + \sqrt{2}\frac{\rho}{1+\delta}\cos\theta) \frac{m_\eta^2 - m_\pi^2}{m_{\eta'}^2 - m_\pi^2} \right], \end{aligned} \quad (\text{A2})$$

where θ is the $\eta - \eta'$ mixing angle and the parameters ρ and δ , defined by

$$\begin{aligned} \langle \eta_8 | H_W | K^0 \rangle &= \sqrt{\frac{1}{3}} (1 + \delta) \langle \pi^0 | H_W | K^0 \rangle, \\ \langle \eta_0 | H_W | K^0 \rangle &= -2\sqrt{\frac{2}{3}} \rho \langle \pi^0 | H_W | K^0 \rangle, \end{aligned} \quad (\text{A3})$$

measure the breakdown of nonet symmetry in $K^0 - \eta_0$ transition and of SU(3)-flavor symmetry in $K^0 - \eta_8$, respectively. We can use the radiative decays $K_L \rightarrow \gamma\gamma$ and $\pi^0 \rightarrow \gamma\gamma$ to constrain ρ and δ :

$$\frac{A(K_L \rightarrow \gamma\gamma)}{A(\pi^0 \rightarrow \gamma\gamma)} = -4 \frac{m_K^2}{m_K^2 - m_\pi^2} \frac{g_8}{f_\pi^2} \zeta, \quad (\text{A4})$$

where

$$\begin{aligned} \zeta = 1 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_\eta^2} & \left(\sqrt{\frac{1}{3}} (1 + \delta) \cos\theta + 2\sqrt{\frac{2}{3}} \rho \sin\theta \right) \left(\sqrt{\frac{1}{3}} \frac{f_\pi}{f_8} \cos\theta - 2\sqrt{\frac{2}{3}} \frac{f_\pi}{f_0} \sin\theta \right) \\ & + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_{\eta'}^2} \left(\sqrt{\frac{1}{3}} (1 + \delta) \sin\theta - 2\sqrt{\frac{2}{3}} \rho \cos\theta \right) \left(\sqrt{\frac{1}{3}} \frac{f_\pi}{f_8} \sin\theta + 2\sqrt{\frac{2}{3}} \frac{f_\pi}{f_0} \cos\theta \right), \end{aligned} \quad (\text{A5})$$

and $g_8 = 0.26 \times 10^{-5} m_K^2$ is the coupling constant in the $\Delta S = 1$ effective chiral Lagrangian. From the data of $K_L \rightarrow \gamma\gamma$ and $\pi^0 \rightarrow \gamma\gamma$, we find $|\zeta| = 0.87$. Using

$$\theta = -15.4^\circ, \quad f_8/f_\pi = 1.26, \quad f_0/f_\pi = 1.17 \quad (\text{A6})$$

determined phenomenologically [30] and $\delta = 0.17$ [31], we obtain $\rho = 0.96$. Numerically, we find that Ω_{IB} is almost insensitive to the values of δ , ρ and θ as long as they are constrained by ζ . Hence, to a very good approximation, we obtain

$$\Omega_{\text{IB}} = (10.45 \pm 0.05) \frac{m_d - m_u}{m_s}, \quad (\text{A7})$$

where the experimental value of $A_0/A_2 = 22.2$ has been used.

B. ANOMALOUS DIMENSIONAL AND CONSTANT MATRICES

For reader's convenience, we list here the anomalous dimensional matrix $\gamma_V^{(0)}$ and the constant matrix \hat{r}_V appearing in Eq. (2.4):

$$\gamma_V^{(0)} = \begin{pmatrix} -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & -2 \end{pmatrix}, \quad (\text{B1})$$

and [3]

$$\hat{r}_V^{\text{NDR}} = \begin{pmatrix} 3 & -9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 17 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 & 17 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9 & 3 \end{pmatrix} \quad (\text{B2})$$

in the NDR scheme, and

$$\hat{r}_V^{\text{HV}} = \begin{pmatrix} \frac{7}{3} & -7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -7 & \frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{7}{3} & -7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -7 & \frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{47}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{47}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{3} & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & \frac{7}{3} \end{pmatrix} \quad (\text{B3})$$

in the HV scheme. Note that the 66 and 88 entries of \hat{r}_V given in [3] are erroneous and have been corrected here.